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TOMOGRAPHIC DETERMINATION OF PARTICLE DISTRIBUTION BY VELOCITIES

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1. Measurement of the fluorescence spectrum of a rarefied medium (of a gas or plasma) allows us to determine one of the most important characteristics of the medium, and namely, the distribution function (DF) of the particles by velocities. In the traditional (single-aspect) formulation, in order to determine the DF the $q(\nu, \mathbf{n})$ spectrum of radiation propagating in the direction $\mathbf{n}$ is recorded. In this case the function $q(\nu, \mathbf{n})$ is associated with the function $f(v, \mathbf{n})$ for the distribution of the particles along the projections of the velocities in the direction of $\mathbf{n}$ by the following equation [1, 2]:

$$\int_{-\infty}^{\infty} K\left(\frac{\nu_0}{c}(v - v')\right) f(v', \mathbf{n})\, dv' = q\left(\nu_0 \frac{v}{c}, \mathbf{n}\right). \tag{1.1}$$

Here $K(\nu)$ is the kernel by means of which we take into consideration the effects of the non-Doppler broadening mechanisms and the equipment function of the spectral instrumentation; $\nu = \nu_0 v/c$; ν_0 is the frequency characterizing the position of the center of the radiation lines; c is the speed of light.

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Based on the $f(v, \mathbf{n})$ of the three-dimensional particle distribution by velocities $F(\mathbf{V})$, where $\mathbf{V} = (V_x, V_y, V_z)$, is impossible in the general case. We can do this only in the special cases in which we introduce a priori assumptions with respect to the angular distribution function (for example, with respect to its isotropicity). In the present paper we examine a more general formulation of the problem, which allows us to measure the DF without resorting to these assumptions.

2. In the absence of a priori information regarding the angular structure of the DF, the function $F(\mathbf{V})$ can be determined from the results of multiaspect spectroscopic observation, i.e., from a set of one-dimensional DF $f(v, \mathbf{n})$ for various orientations of the vector

$$\mathbf{n} = (\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta), \tag{2.1}$$

where θ and φ denote the polar and azimuthal angles of observation in a spherical coordinate system. Following [3], we will write the relationship between $f(v, \mathbf{n})$ and $F(\mathbf{V})$ in the form

$$\int F(\mathbf{V})\,\delta(v - \mathbf{n}\cdot\mathbf{V})\,d^3\mathbf{V} = f(v, \mathbf{n}). \tag{2.2}$$

Equation (2.2) is the Radon transform of the function $F(\mathbf{V})$ in the three-dimensional space of velocities, where for each vector $\mathbf{n}$ (for each set of angles θ, φ) it is necessary to solve Eq. (1.1) in advance.

The question of finding a numerical solution for an equation of the type of (1.1) has been dealt with rather fully in [4]. Below we undertake a study of the possibilities for a numerical solution of Eq. (2.2). We have developed a numerical algorithm for the inversion of (2.2), which is based on the Fourier-analysis method with respect to the projection data of $f(v, \mathbf{n})$ [5].

3. The three-dimensional Fourier transform of the function $F(\mathbf{V})$, written in a spherical coordinate system, is denoted $\hat{F}(k\mathbf{n}) = \hat{F}(k\sin\theta\cos\varphi,\ k\sin\theta\sin\varphi,\ k\cos\theta)$. According to the theorem of the central section familiar in computational tomography, we are dealing with the relationship [5]

$$\widehat{F}(k\mathbf{n}) = \widehat{f}(k, \mathbf{n}) = \int_{-\infty}^{\infty} f(v, \mathbf{n})\exp(-2\pi ikv)\,dv. \tag{3.1}$$

Expression (3.1) means that the one-dimensional Fourier transform of the function $f(v, \mathbf{n})$ with respect to the variable v is equal to the three-dimensional Fourier transform of $\hat{F}(k\mathbf{n})$ on a ray with direction $\mathbf{n}(\theta, \varphi)$. In the numerical execution of the Fourier synthesis method a Fourier transform $\hat{F}(\mathbf{k})$ of the function $F(\mathbf{V})$ is taken from the set of functions $f(v, \mathbf{n})$ on a discrete grid in a spherical coordinate system, and then the spectrum $\hat{F}(\mathbf{k})$ is recalculated for the grid specified in the Cartesian coordinate system $\mathbf{k} = (k_x, k_y, k_z)$.

To make the transition from the derived Fourier transform $\hat{F}(\mathbf{k})$ to the DF of $F(\mathbf{V})$ we have to carry out the inverse three-dimensional Fourier transformation

$$F(\mathbf{V}) = \int \widehat{F}(\mathbf{k})\exp(2\pi i\mathbf{k}\cdot\mathbf{V})\,d^3\mathbf{k}. \tag{3.2}$$

Under realistic conditions $f(v, \mathbf{n})$ contains random noise and, consequently, $\hat{F}(\mathbf{k})$ contains high-frequency harmonics which are converted by the Fourier transformation operation (3.2) into a numerically unstable procedure. To prevent these difficulties, in the place of the function $\hat{F}(\mathbf{k})$ we construct its regularized analog [6]

$$\widehat{F}_\alpha(\mathbf{k}) = \widehat{F}(\mathbf{k}) \Big/ \left(1 + \alpha\sum_{i=0}^{l} d_i\,(k_x^2 + k_y^2 + k_z^2)^i\right),$$

where α is the regularization parameter; ℓ is the regularization order; $d_i \geq 0$ are weighting factors. The choice of ℓ and d_i are determined by a priori data on the smoothness of the sought solution.

As a result of the inverse Fourier transformation we obtain a regularized estimate for $F(\mathbf{V})$:

$$F_\alpha(\mathbf{V}) = \int \widehat{F}_\alpha(\mathbf{k})\exp(2\pi i\mathbf{k}\cdot\mathbf{V})\,d^3\mathbf{k}.$$

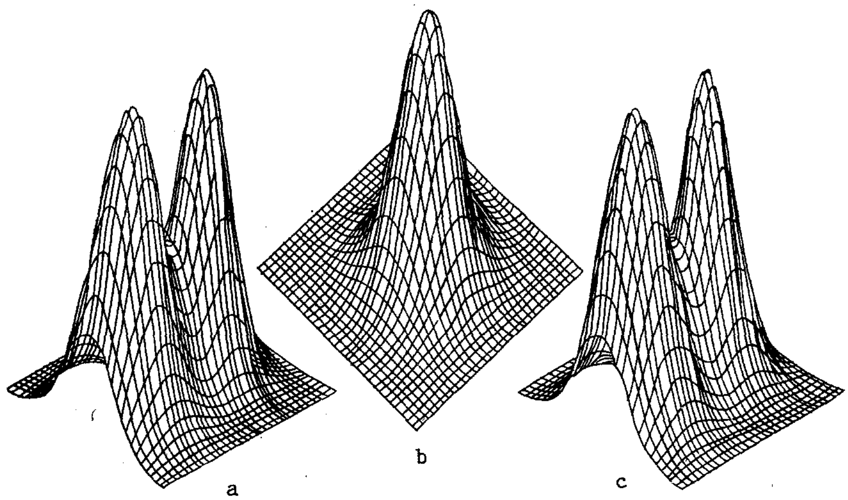

Fig. 1

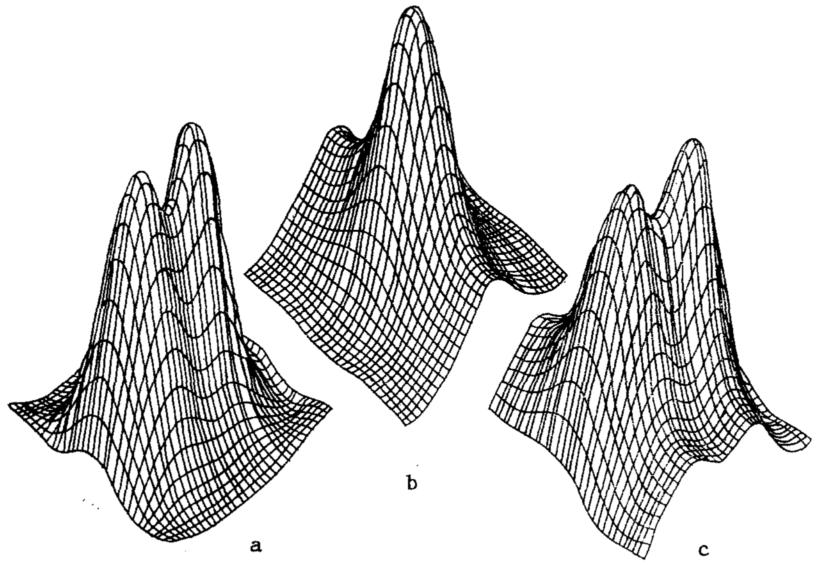

Fig. 2

The selection of the regularization parameter α is based on the condition of agreement between the nonclosure and the noise level in the experimental data. A method for the selection of α that is based on the functional probability maximum [4] is used in this study.

4. The algorithm has been tested numerically on a model of the problem. For our model distribution we have taken the function

$$F(\mathbf{V}) = \sum_{i=1}^{2} a_i \exp\left(-b_i^2 |\mathbf{V} + \mathbf{u}_i|^2\right) \tag{4.1}$$

where $a_1 = 1$, $a_2 = 1$, $b_1 = 4$, $b_2 = 5$, $\mathbf{u}_1 = (0, 0.25, 0)$, $\mathbf{u}_2 = (0; -0.15, 0)$. Test function (4.1) may be regarded as a mathematical model of the DF of a nonequilibrium two-component medium.

Transform (2.2) of function (4.1) has the form

$$f(v; \mathbf{n}) = \pi \sum_{i=1}^{2} \frac{a_i}{b_i^2} \exp\left(-b_i^2 (v + \mathbf{u}_i \cdot \mathbf{n})^2\right),$$

where the vector $\mathbf{n}$ is determined through relationship (2.1).

The numerical calculations were carried out with a uniform grid for the variable v with $N_V = 32$ readings. The number of checks on the angles θ and φ was chosen, respectively, to equal N_θ and N_φ ($\theta \in [0, \pi/2], \varphi \in [0, 2\pi]$) with a uniform interval for θ and φ. The noise level of the function $f(v, \mathbf{n})$ was taken to be equal to 1% of the maximum for $f(v, \mathbf{n})$.

Figures 1 and 2 illustrate the calculation results. The isometric results shown in Fig. 1 represent the exact distribution (4.1) in the sections $V_x = 0$ (a), $V_y = 0$ (b), and $V_z = 0$ (c). In Fig. 2 we see the corresponding sections for the reproduced distribution $F_\alpha(\mathbf{V})$ with $N_\theta = 10$, $N_\varphi = 10$.

The results of the numerical calculations demonstrate the possibility of finding a three-dimensional distribution of particles by velocity through the means of computational tomography.

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A NUMERICAL AND EXPERIMENTAL STUDY OF MULTICASCADE INDUCTION ACCELERATOR OF CONDUCTORS

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In a number of branches of sciences and engineering it is necessary to develop high speeds for the motion of solids, and these can be achieved by placing conductors into powerful magnetic fields. An effective means of accomplishing this with high speeds is the acceleration of plane annular conductors in a pulsed magnetic field generated by a plane annular inductor [1]. However, it sometimes is necessary to accelerate three-dimensional bodies, in particular, those that are cylindrical in shape. Such conductors can be accelerated in a pulsed magnetic field generated by the inductor in the form of a solenoid coil. Multicascade accelerators of conductors can be based on the inductor system of the solenoid type, and these make it possible to achieve high velocities with limited mechanical load on the body being accelerated.

The articles in [2-4] are devoted to a theoretical study of the electromechanical processes encountered in single-cascade accelerators (containing a single acceleration coil) with a solenoid-type inductor to which power is supplied from a capacitor battery. A mathematical model of a solenoid-type inductor system has been developed in [2] involving the utilization of a method of integral equations, and where the existence of an optimum mass

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